note: if there is no numerator, obtaining the SSR becomes easy.

Ex:

$$\frac{Y(s)}{U(s)} = \frac{1}{s^4 + 3s^3 + 2s^2 + s + 1}$$

$$\mathcal{L}^{-1} \Rightarrow \quad y^{(4)} + 3y^{(3)} + 2\ddot{y} + \dot{y} + y + 1 = u$$

$$\begin{aligned} x_1 &= y & \dot{x}_1 &= x_2 \\ x_2 &= \dot{y} & \dot{x}_2 &= x_3 \\ x_3 &= \ddot{y} & \dot{x}_3 &= x_4 \\ x_4 &= y^{(3)} & \dot{x}_4 &= y^{(4)} \end{aligned}$$

$$\begin{aligned} y^{(4)} &= -3y^{(3} - 2\ddot{y} - \dot{y} - y + u \\ &= -x_1 - x_2 - 2x_3 - 3x_4 + u \end{aligned}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = x_1$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad D = 0$$

$$\frac{Y(s)}{U(s)} = \frac{s+2}{s^2+3s+4}$$

makes it impossible to do SSR like before.

$$\mathcal{L}^{-1} \Rightarrow \ddot{y} + 3\dot{y} + 4y = \boxed{\dot{u}} + 2u$$

note: It is not possible to obtain SSR in the same fashion.

$$\therefore \quad \frac{Q(s)}{U(s)} = \frac{1}{s^2+3s+4} \qquad ①$$

$$\frac{Y(s)}{Q(s)} = s+2 \qquad ②$$

$$① \rightarrow \mathcal{L}^{-1} \Rightarrow \quad \ddot{q}_o + 3\dot{q}_o + 4q_o = u$$

$$x_1 = q_o \qquad \dot{x}_1 = x_2$$

$$x_2 = \dot{q}_o \qquad \dot{x}_2 = \ddot{q}_o$$

$$\ddot{q}_o = -3\dot{q}_o - 4q_o + u$$

$$= -3x_2 - x_1 + u$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$② \rightarrow \mathcal{L}^{-1} \Rightarrow \quad y = \dot{q}_o + 2q_o$$

$$= 2x_1 + x_2$$

$$c = \begin{bmatrix} 2 & 1 \end{bmatrix} \qquad D = 0$$

because the degree of numerator is less the denomator

EX:

$$\frac{Y(s)}{u(s)} = \frac{2s^2 + 3s + 4}{s^2 + 4s + 1}$$

$$= \frac{2[s^2 + 4s + 1] - 8s - 2 + 3s + 4}{s^2 + 4s + 1}$$

$$= \boxed{2} + \frac{-5s + 2}{s^2 + 4s + 1}$$

$B_0 = D$

$$\frac{Q(s)}{u(s)} = \frac{1}{s^2 + 4s + 1}$$

$$u = \ddot{q}_0 + 4\dot{q}_0 + q_0$$

$x_1 = q_0 \qquad \dot{x}_1 = x_2$

$x_2 = \dot{q} \qquad \dot{x}_2 = \ddot{q}_0$

$$\ddot{q}_0 = -4\dot{q}_0 - q_0 + u$$
$$= -x_1 - 4x_2 + u$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$D = 2 \qquad C = [\,2 \quad -5\,]$$

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}$$

CONTROLABLE CANONICAL FORM

$$\dot{x} = A_c x + B_c u$$
$$y = C_c x + D_c u$$

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & & & & \vdots \\ \vdots & 0 & 0 & 1 & \ddots & & & & \vdots \\ \vdots & 0 & 0 & 0 & \ddots & 0 & & & 0 \\ & & & & & 0 & 1 & 0 & 0 \\ 0 & & & & & 0 & 0 & 1 & 0 \\ 0 & & & & & & 0 & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & & & & -a_{n+1} & & -a_n \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_c = [b_n - a_n b_0, b_{n-1} - a_{n-1} b_0, \ldots, b_1 - a_1 b_0]$$
$$D_c = b_0$$

OBSERABLE CANONICAL FORM

$$\dot{x} = A_o x + B_o u$$
$$y = C_o x + D_o u$$

$$A_o = A_c^T$$
$$B_o = C_c^T$$
$$C_o = B_c^T$$
$$D_o = D_c$$

DIAGONAL CANONICAL FORM

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s + b_n}{(s+P_1)(s+P_2)\ldots(s+P_n)}$$

$P_1, P_2, \ldots, P_n$ are real and distinct.

$$\frac{Y(s)}{U(s)} = b_0 + \frac{C_1}{s+P_1} + \frac{C_2}{s+P_2} + \ldots + \frac{C_n}{s+P_n}$$

$C_1, C_2, \ldots, C_n$ vector C.

$$Y(s) = b_0 U(s) + \underbrace{\frac{C_1}{s+P_1} U(s)}_{x_1} + \underbrace{\frac{C_2}{s+P_2} U(s)}_{x_2} + \ldots + \underbrace{\frac{C_n}{s+P_n} U(s)}_{x_n}$$

$$x_1 = \frac{U(s)}{s+P_1} \qquad \dot{x}_1 = -P_1 x_1 + u$$

$$x_2 = \frac{U(s)}{s+P_2} \qquad \dot{x}_2 = -P_2 x_2 + u$$

$$x_n = \frac{U(s)}{s+P_n} \qquad \dot{x}_n = -P_n x_n + u$$

$$A = \begin{bmatrix} -P_1 & 0 & \cdots & 0 \\ 0 & -P_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & -P_n \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$C = [C_1, C_2 \ldots C_n] \qquad D = b_0$$

JORDON CANONICAL FORM

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \dots + b_{n-1} s + b_n}{(s+P_1)^3 (s+P_4) \dots\dots (s+P_n)}$$

$P_1, P_4, \dots, P_n$ are real distinct roots.

$$\frac{Y(s)}{U(s)} = b_0 + \frac{C_1}{(s+P_1)^3} + \frac{C_2}{(s+P_1)^2} + \frac{C_3}{(s+P_1)} + \frac{C_4}{(s+P_4)} + \dots + \frac{C_n}{(s+P_n)}$$

Jordon block

$$A = \begin{bmatrix} \boxed{\begin{matrix} -P_1 & 1 & 0 \\ 0 & -P_1 & 1 \\ 0 & 0 & -P_1 \end{matrix}} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix} \\ \begin{matrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} -P_4 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & P_n \end{matrix} \end{bmatrix} \qquad B = \begin{bmatrix} \boxed{\begin{matrix} 0 \\ 0 \end{matrix}} \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

represents repeatations

$$C = [C_1 \;\; C_2 \;\; \cdots \;\; C_n] \qquad D = b_0$$

consider

$$Y(s) = b_0 U(s) + \frac{C_1 U(s)}{(s+P_1)^3} + \frac{C_2 U(s)}{(s+P_1)^2} + \frac{C_3 U(s)}{(s+P_1)} + \frac{C_4 U(s)}{s+P}$$

STATE SPACE REPERSENTATIONS IN CANONICAL FORM

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1}\dot{y} + a_n y = b_0 u^{(n)} + b u^{(n-1)} + \dots + b_n u$$

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^{(n)} + b s^{(n-1)} + \dots + b_{n-1} s^2 + b_n s}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$

note: the degree of the polynomial on the top must be less then or equal to that on the bottom.

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ & 0 & & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$D = b_0 \qquad C = \left[b_n - a_n b_0 ,\ b_{n-1} - a_{n-1} b_0 ,\ \cdots ,\ b_1 - a_1 b_0 \right]$$

note: $b_0 = 0$ then $D = 0$, the only time that we have a D matrix is when there are the same order of numerator and denomadator.

EX:

$$G(s) = \frac{2s^2+3s+4}{s^2+s+1}$$

$$= \frac{[2s^2+2s+2]+s+2}{s^2+s+1}$$

$$\frac{Y(s)}{U(s)} = 2 + \frac{s+2}{s^2+s+1}$$

$$Y(s) = 2U(s) + \underbrace{\boxed{\frac{s+2}{s^2+s+1}U(s)}}_{\hat{y}(s)}$$

$$\hat{y}(s) = \frac{s+2}{s^2+s+1}U(s)$$

$$\frac{Q(s)}{U(s)} = \frac{1}{s^2+s+1}$$

$$\frac{\hat{Y}(s)}{Q(s)} = s+2$$

$$Q(s)(s^2+s+1) = U(s)$$

$$\ddot{q} + \dot{q} + q = u$$

$$x_1 = q$$

$$x_2 = \dot{q}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{q} = \ddot{x}_1$$